Analysis of Lossy Transmission Lines with Arbitrary Nonlinear Terminal Networks

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Abstract —A novel method for transient analysis of lossy transmission lines with arbitrary nonlinear terminal networks is presented. The uniqueness of this approach is that we develop time-domain Green's functions for the multiport transmission-line systems by terminating the ports in quasi-matched loads. This ensures Green's functions of a short duration. Hence, the amount of frequency-domain data necessary to obtain time-domain Green's functions is modest. These Green's functions are then convolved with the line port voltages. With this technique one can analyze responses of multiconductor transmission lines with arbitrary nonlinear loads (even with memory) as we have at any instant of time Thévenin's equivalent of the linear portion of the system. An example is presented to illustrate the application of this technique to multiconductor nonlinearly loaded transmission lines.

I. Introduction

NONLINEAR EFFECTS in multiconductor transmission line systems are important when there are semiconductor devices, like diodes and transistors, voltage limiters, and so on, connected to the transmission lines. Nonlinearities become important when a device is changing its state and/or when it is excited by a large-amplitude signal.

Multiconductor transmission lines have been analyzed either by a direct time-domain approach or by transforming frequency-domain information [1]–[12]. However, the analysis of lossy lines is possible only in the frequency domain. Liu and Tesche [13] have presented two methods for analyzing linear electromagnetic systems (in particular, antennas or scatterers) with nonlinear loads. Their first method is a direct time-domain approach which involves solving a space-time domain integral equation. The second method is a technique for obtaining the response of the antenna by making use of frequency-domain data, such as the short-circuit current and the driving-point admittance, both being solutions to the linear problem in the absence of nonlinearities. Then they solve the nonlinear problem by time-stepping and convolution utilizing the solution to the linear portion of the system.

In this paper, the second technique presented in [13] has been extended and applied to the analysis of nonlinearly loaded transmission lines. We first obtain frequency-domain Y-parameters of the linear portion of the lossy multiconductor transmission lines by utilizing the modal analysis in frequency domain. However, our approach differs from that in [13] as we find the equivalent parameters of a suitably terminated (quasi-matched) multiconductor transmission line, instead of the short-circuited line. This procedure substantially reduces the amount of computations required to obtain the final solution. Next, we perform the inverse Fourier transform of the Y-parameters to obtain time-domain Green's functions, i.e., the responses of the terminated line due to impulse excitations. Finally, we consider the line with the nonlinear terminal networks and convolve Green's functions with the voltages at the line ports to obtain a time-stepping solution for the port voltages and currents. The method is suitable for arbitrary terminal networks, as we need not recompute frequency-domain data if we change the excitation waveform or any other characteristic of the terminal networks.

II. Theory

The analysis of arbitrary nonlinear terminal networks (with or without memory), in the general case, can be performed only in time domain. On the other hand, the analysis of lossy transmission lines (as well as lines with frequency-dependent parameters) can be performed only in frequency domain. So, in order to combine the two cases, i.e., to design a method for analysis of lossy transmission lines with arbitrary nonlinear terminal networks, one must be able to combine the solutions in the two domains. Since the transmission line is a linear network, it can be characterized completely in time domain by its Green's functions, which are, in turn, obtained from the frequency-domain analysis. These functions can be implemented in a time-domain solution of the terminal networks in a manner shown below.

Consider a linear, passive n-port network. Suppose that an ideal voltage generator, of emf $v_{j0}(t)$, is connected at the port j, while the other ports are short-circuited. One can solve for the currents at the network ports. All these currents can be represented in the form

$$I_k(\omega) = Y_{kj}(\omega) V_{j0}(\omega), \qquad k = 1, \cdots, n \tag{1}$$

where $V_{j0}(\omega)$ is the Fourier transform of $v_{j0}(t)$ and $Y_{kj}(\omega)$ are the network Y-parameters, while ω is the angular frequency. Let us suppose, for a moment, that $v_{j0}(t)$ is a

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unit delta function. In that case, $V_{j0}(\omega) = 1$, independently of frequency, and the currents in the time domain are obtained as

$$i_k(t) = i_{gkj}(t) = F^{-1}\{Y_{kj}(\omega)\} \tag{2}$$

where F^{-1} denotes the inverse Fourier transform. These currents are referred to as the network Green's functions. (There are two things to be noted. First, the reference direction for the generator emf and the current at that port coincide, by convention. Second, if the network is reciprocal, as in our case, then $i_{gkj}(t) = i_{gjk}(t)$.)

Let us go back to the case when $v_{j0}(t)$ is an arbitrary function. Now we have

$$i_k(t) = F^{-1}\{Y_{kj}(\omega)V_{j0}(\omega)\} = i_{gkj}(t) * v_{j0}(t) \tag{3}$$

where "$*$" denotes the convolution. By the superposition principle, which is valid for linear networks, if we now consider the same network driven by ideal voltage generators at all the ports, we can write

$$i_k(t) = \sum_{j=1}^{n} \int_0^t i_{gkj}(t-\tau) v_{j0}(\tau) d\tau \tag{4}$$

where the convolution is represented by its integral form assuming that all the excitations begin after $t = 0$. It should be noted that, by the compensation theorem, the ideal voltage generators driving the network can be considered as a substitution of the outside circuitry connected to the network. Now, the emf's of these generators must equal the voltages at the network ports.

Following the above approach, we would have to connect an ideal delta-function generator between one of the transmission line conductors (at one line end) and ground, short-circuit all other line ports, perform the modal analysis in the frequency domain to find the conductor currents, and compute the inverse Fourier transform to obtain Green's functions. This should be repeated for all line conductors.

There are, however, several problems that should be considered. First, the analysis of the transmission line is usually done only numerically, at a finite number of discrete frequencies. In turn, in time domain, Green's functions also must be discretized and of finite duration. Second, these Green's functions must be convolved with line port voltages, which also has to be done numerically. The convolution turns out to be the most time-consuming process in the present analysis. Therefore, the number of samples of Green's functions should be kept as low as possible. This can be a particular problem if the analysis of the response of the line with terminal networks is to span a time interval greater than a few line transit times. Namely, if the line ports are short-circuited (as they are for the computation of Green's functions), the line response exceeds many transit times in duration, even for a moderately lossy line. For a lossless line with short-circuited ports, the response is of infinite duration! Therefore, the line Green's functions would have to be kept in very long registers, spanning the same time interval as the time interval in which we would like to analyze the response of the transmission line terminated by arbitrary nonlinear networks. This is, of course, not only a computer-storage problem, but also demands very long execution times.

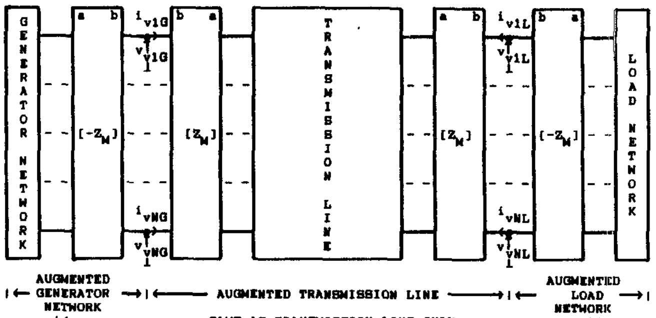

Fig. 1. Schematic representation of lossy multiconductor transmission line with arbitrary terminal networks and auxiliary networks $[Z_M]$ and $[-Z_M]$.

The lengths of the registers mentioned above could be kept relatively short if the duration of Green's functions could be reduced to only a few line transit times. However, such a situation is possible only if the line is reasonably well matched. As an example, let us consider a lossless line terminated in perfectly matched networks and excited by a delta-function generator at one line port. The duration of the line response is only one sample for all the ports at the same line end where the generator is connected. The response at the ports at the other line end terminates after one line transit time. For lossy lines, with moderately low losses (as normally used in practice), the situation is similar.

Following the above example, we would like to have a well-matched transmission line when computing Green's functions. However, later on we have to be able to use these Green's functions to obtain the response of the line terminated in given terminal networks. To achieve both goals, we can artificially insert between the transmission line and the terminal networks two pairs of passive networks, denoted as $[Z_M]$ and $[-Z_M]$ in Fig. 1. The transmission line with the two $[Z_M]$ networks we shall refer to as the augmented transmission line, while the terminal networks with the $[-Z_M]$ networks we shall refer to as the augmented terminal networks. Our objective is to synthesize the networks $[Z_M]$ and $[-Z_M]$ so as to minimize the duration of the augmented transmission line Green's functions, yet to make the series combination of the $[Z_M]$ and $[-Z_M]$ networks behave as a set of short circuits between the transmission line and the terminal networks.

Let the inserted networks have $2N$ ports, where N is the number of line signal conductors. Let us also denote one set of N ports as the side "a" of the network, and the other N ports as the side "b" of the network, as shown in Fig. 1. Let us represent the Z-matrix of the network $[Z_M]$ as

$$[Z_M] = \begin{bmatrix} [Z_a] & [Z_m] \\ [Z_m] & [Z_b] \end{bmatrix} \tag{5}$$

where the blocks $[Z_a]$ and $[Z_b]$ shall be referred to as the self blocks, and the block $[Z_m]$ as the mutual block of

Z-parameters. Let the network $[-Z_M]$ consist of the same elements as the network $[Z_M]$, but of opposite signs. Then

$$[-Z_M] = -[Z_M]. \tag{6}$$

In addition, we shall refer to the ports at the "b" sides of the networks $[Z_M]$ and $[-Z_M]$ as the virtual ports, and we shall refer to the voltages and currents at these ports as the virtual port voltages, namely currents.

We have now to find $[Z_a]$, $[Z_b]$, and $[Z_m]$ so that the augmented transmission line is well matched (i.e., quasi-matched) when the ports at the side "b" of the network $[Z_M]$ are short circuited to ground; e.g., if the transmission line is assumed to have a real and frequency-independent characteristic impedance matrix $[Z_c]$, we have to fulfil the condition

$$[Z_c] = [Z_a] - [Z_m][Z_b]^{-1}[Z_m]. \tag{7}$$

Of course, there are many equivalent realizations of resistive networks that satisfy (7).

Once we have designed the network $[Z_M]$, we automatically know the network $[-Z_M]$. It can be easily shown that the cascade of these two networks behaves as a set of short circuits between the corresponding ports at the sides "a" of the networks $[Z_M]$ and $[-Z_M]$ (see Appendix).

Considering now the augmented transmission line as a network with $n = 2N$ ports, we can determine its Green's functions. These functions known, we can relate the virtual port currents $i_{vk}(t)$ to the virtual port voltages $f_{vj}(t)$ by using (4), where $v_{j0}(t)$ should be substituted by $v_{vj}(t)$. In order to distinguish between the line ports at the generator and at the load end, we can introduce indices "G" for the quantities corresponding to the generator end, and "L" for the load end, and rewrite (4) as

$$i_{vkG}(t) = \sum_{j=1}^{N} \int_0^t i^s_{gkj}(t-\tau) v_{vjG}(\tau)\, d\tau + \sum_{j=1}^{N} \int_0^t i^m_{gkj}(t-\tau) v_{vjL}(\tau)\, d\tau, \qquad k = 1, \cdots, N \tag{8}$$

$$i_{vkL}(t) = \sum_{j=1}^{N} \int_0^t i^m_{gkj}(t-\tau) v_{vjG}(\tau)\, d\tau + \sum_{j=1}^{N} \int_0^t i^s_{gkj}(t-\tau) v_{vjL}(\tau)\, d\tau, \qquad k = 1, \cdots, N. \tag{9}$$

In these equations, i^s_{gkj} is Green's function representing the current at the virtual port k when the delta-function generator drives the virtual port j at the same line end, while i^m_{gkj} corresponds to the case when the current is computed at one line end, while the excitation is at the other end. Obviously, due to the symmetry of the transmission line, it is irrelevant which end of the line is taken as the first and which as the second one.

In order to prepare (8) and (9) for computer use, we have to replace the integrations by summations. Thus we obtain

$$i_{vkG}(q) = \sum_{j=1}^{N} \sum_{p=0}^{q} i^s_{gkj}(q-p) v_{vjG}(p) \Delta t + \sum_{j=1}^{N} \sum_{p=0}^{q} i^m_{gkj}(q-p) v_{vjL}(p) \Delta t, \qquad k = 1, \cdots, N \tag{10}$$

$$i_{vkL}(q) = \sum_{j=1}^{N} \sum_{p=0}^{q} i^m_{gkj}(q-p) v_{vjG}(p) \Delta t + \sum_{j=1}^{N} \sum_{q=0}^{q} i^s_{gkj}(q-p) v_{vjL}(p) \Delta t, \qquad k = 1, \cdots, N \tag{11}$$

where the argument (q) denotes the time instant $q\Delta t$ at which we take the voltages and currents. We can modify the sums on the right-hand sides of (10) and (11) by extracting the terms for $p = q$. Noting that $i^s_{gkj}(0) \neq 0$, and $i^m_{gkj}(0) = 0$ (due to the line delay), we have

$$i_{vkG}(q) = \sum_{j=1}^{N} i^s_{gkj}(0) v_{vjG}(q) \Delta t + \sum_{j=1}^{N} \sum_{p=0}^{q-1} i^s_{gkj}(q-p) v_{vjG}(p) \Delta t + \sum_{j=1}^{N} \sum_{p=0}^{q-1} i^m_{gkj}(q-p) v_{vjL}(p) \Delta t, \qquad k = 1, \cdots, N \tag{12}$$

$$i_{vkL}(q) = \sum_{j=1}^{N} i^s_{gkj}(0) v_{vjL}(q) \Delta t + \sum_{j=1}^{N} \sum_{p=0}^{q-1} i^m_{gkj}(q-p) v_{vjG}(p) \Delta t + \sum_{j=1}^{N} \sum_{p=0}^{q-1} i^s_{gkj}(q-p) v_{vjL}(p) \Delta t, \qquad k = 1, \cdots, N. \tag{13}$$

Note that the first sum in either of (12) and (13) contains virtual voltages only for $t = q\Delta t$, i.e., at the same time instant for which the current on the left-hand side is computed, while the second (double) sum contains only the previous values of the voltages, i.e., the history of the network. Noting that $i^s_{gkj}(0)$ are constants for a given transmission line, the first sum can be represented for $k = 1, \cdots, N$ in the form $[G_{vd}][v_v]$, where $[v_v]$ is a column matrix containing the virtual voltages, and $[G_{vd}]$ is a $N \times N$ square matrix, the elements of which are $i^s_{gkj}(0)$. The matrix $[G_{vd}]$ can be considered as a conductance matrix giving the instantaneous (dynamic) input conductance to the transmission line as seen from the virtual ports. The double sum represents a current. It can be considered as a current of an independent current generator, the current of which does not depend on the instantaneous values of the transmission line currents and voltages, but rather only on their previous values. Again, if we consider $k = 1, \cdots, N$, these independent currents can be represented by a column matrix $[i_c]$, where the subscript "c" points out that these currents are obtained by convolving Green's functions with the virtual port voltages. Thus, (12) and (13) can be written in a shorter form

$$[i_{vG}(q)] = [G_{vd}][v_{vG}(q)] + [i_{cG}(q-1)] \quad (14)$$

$$[i_{vL}(q)] = [G_{vd}][v_{vL}(q)] + [i_{cL}(q-1)] \quad (15)$$

where $[i_{vG}]$ and $[i_{vL}]$ are column matrices containing the virtual port currents. We can now solve (14) and (15) for the virtual voltages at $t = q\Delta t$ to obtain

$$[v_{vG}(q)] = [G_{vd}]^{-1}[i_{vG}(q)] - [G_{vd}]^{-1}[i_{cG}(q-1)] \quad (16)$$

$$[v_{vL}(q)] = [G_{vd}]^{-1}[i_{vL}(q)] - [G_{vd}]^{-1}[i_{cL}(q-1)]. \quad (17)$$

There are, however, certain problems that have to be considered. First, any real transmission line has frequency-dependent parameters, i.e., its characteristic impedance cannot be represented by a purely resistive network. We have to notice that the augmented terminal networks comprise the networks $[-Z_M]$. Since the analysis of the terminal networks is to be performed in time domain, it is not possible to model the network $[Z_M]$ by frequency-dependent elements. This means that we cannot make a perfect match for a real (lossy) transmission line. However, this should not be a serious problem, because the characteristic impedance matrix of a line with relatively low losses does not significantly depend on frequency. Furthermore, this matrix is almost real, and very close to the characteristic impedance matrix of a lossless line that has the same inductance and capacitance matrices as the lossy line under consideration. Second, the network $[Z_M]$ should be as simple as possible. and it is advisable that its elements are pure resistances, so that the analysis of the terminal networks does not get too involved.

The simplest choice of the network $[Z_M]$ is to take simple resistors and connect them between the corresponding ports at the sides "a" and "b". In order to obtain a reasonably good match, the resistances can be taken equal to the corresponding diagonal elements of the characteristic impedance matrix of the corresponding lossless line $[Z_c]$, i.e., Z_{ckk}. Thereby, in practical cases of lossy lines, the response of the augmented transmission line (when computing Green's functions) is confined to about 3–6 line transit times, and Green's function registers have to cover only this time span. With these simple resistance matrices, our system looks as shown in Fig. 2. Of course, if the coupling between the adjacent conductors is extremely large, as, for example, in certain filters, a more sophisticated network $[Z_M]$ might be needed.

If we now connect the terminal networks augmented by the negative resistances to the augmented transmission line (see Fig. 2), the conductor currents and the voltages between the junctions of Z_{ckk} and $-Z_{ckk}$ and the ground are related by (16) and (17) where the subscript "v" for the currents can be omitted. Note that the series combination of Z_{ckk} and $-Z_{ckk}$ essentially represents a mere short circuit. The terminal voltages at the real transmission line

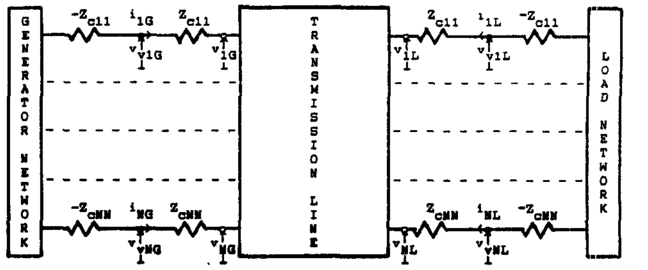

Fig. 2. Schematic representation of the system of Fig. 1, but with the auxiliary networks in the form of a set of resistors.

ports can now be obtained as

$$\begin{aligned}[v_G(q)] &= [v_{vG}(q)] + \mathrm{diag}(-Z_c)[i_G(q)] \\ &= [R_d][i_G(q)] - [G_{vd}]^{-1}[i_{cG}(q-1)] \quad (18)\end{aligned}$$

$$\begin{aligned}[v_L(q)] &= [v_{vL}(q)] + \mathrm{diag}(-Z_c)[i_L(q)] \\ &= [R_d][i_L(q)] - [G_{vd}]^{-1}[i_{cL}(q-1)] \quad (19)\end{aligned}$$

where $\mathrm{diag}(-Z_c)$ is a diagonal matrix the elements of which are $-Z_{ckk}$

$$[R_d] = [G_{vd}]^{-1} + \mathrm{diag}(-Z_c) \quad (20)$$

is the dynamic input-resistance matrix of the line, as seen from the terminal networks, while the term $-[G_{vd}]^{-1}[i_c]$ can be considered as the line open-circuit voltage vector. Hence, we have managed to obtain line equivalent instantaneous Z-parameters (i.e., parameters of the Thévenin equivalent circuit), as seen by the (nonaugmented) terminal networks. It is worth noting that the dynamic input resistance is time constant. In fact, for a lossy line with frequency-independent matrices $[L]$ and $[C]$, $[R_d]$ equals the characteristic impedance matrix of the corresponding lossless line; e.g., for the simplest case of $N=1$, $R_d = \sqrt{L/C}$. This can easily be understood if we take a look at the characteristic impedance matrix in the complex domain. For $N=1$ we have $Z_c = \sqrt{(R+j\omega L)/(G+j\omega C)}$, where R, L, G, and C are line resistance, inductance, conductance, and capacitance per unit length, respectively. At very high frequencies, the imaginary parts of the numerator and denominator dominate over the real parts so that we have $Z_c \approx \sqrt{L/C}$. On the other hand, the dynamic (instantaneous) resistance describes the line behavior for an abruptly changing signal (theoretically, changing instantaneously), for which case the highest frequency components are important for obtaining the time waveform. However, we have discretized the signals in time and applied numerical techniques, thus limiting the spectrum. Therefore, our numerically obtained $[R_d]$ should not exactly coincide with the characteristic impedance of a lossless line. Nevertheless, numerical results have shown that these two matrices have very close elements. The higher the upper frequency limit used in the frequency-domain analysis, the closer are these elements.

Finally, the insertion of negative resistances into the terminal networks can raise a question about the stability of the solution due to numerical errors. Fortunately, there are always some losses present in the transmission line and, usually, in the terminal networks. These losses were found to be sufficient to make the numerical errors invisible in any practical case.

III. Numerical Example

The present method was first checked by comparing the results with other techniques, such as modal analysis in the time domain, modal analysis in the frequency domain, and time-stepping solution of a ladder-network approximation of the transmission line. The comparisons were made for the cases that can be handled by these techniques, e.g., a lossless line with a nonlinear resistive termination, or a lossy line with a linear resistive termination. In all the cases, a good agreement was observed, which was typically within a few percent. Of course, one has to be careful in choosing the time step in the convolution so as to properly sample the waveforms and avoid the aliasing error associated with the fast Fourier transform. However, a comparative analysis would take too much space, and therefore, it is not going to be presented here.

As an example of results obtained by the present technique, we are going to consider a three-conductor lossy transmission line (i.e., $N = 2$). The line length is assumed to be 0.5 m, and the inductance, capacitance, resistance, and conductance matrices at 1 MHz are

$$[L] = \begin{bmatrix} 309 & 21.7 \\ 21.7 & 309 \end{bmatrix} \text{nH/m}$$

$$[C] = \begin{bmatrix} 144 & -6.4 \\ -6.4 & 144 \end{bmatrix} \text{pF/m}$$

$$[R] = \begin{bmatrix} 524 & 33.9 \\ 33.9 & 524 \end{bmatrix} \text{m}\Omega/\text{m}$$

$$[G] = \begin{bmatrix} 905 & -11.8 \\ -11.8 & 905 \end{bmatrix} \text{nS/m.}$$

The resistances were assumed to vary proportionally to the square root of frequency, and the conductances proportionally to the first power of frequency. At one line end, one conductor is driven by a 50-Ω voltage generator (see Fig. 3), of EMF $e(t)$ shown in Fig. 4, while the other conductor is terminated in a 75-Ω load. At the other end, the line is terminated in two nonlinear resistive circuits, each of them being a series combination of a 10-Ω resistor and a nonlinear resistor. The characteristics of the nonlinear resistors were assumed to be given by the equation

$$i_n = 10\left(\exp\left(\frac{v_n}{V_T}\right) - 1\right) \text{ nA} \tag{21}$$

where i_n is the current through the nonlinear resistor, v_n is the voltage at the nonlinear resistor, and $V_T = 25$ mV. Now at each time step, we have to solve (18) and (19) simultaneously with the equations for the terminal networks

$$[v_G(q)] = \begin{bmatrix} e(q\Delta t) \\ 0 \end{bmatrix} - \begin{bmatrix} 50\ \Omega & 0 \\ 0 & 75\ \Omega \end{bmatrix} [i_G(q)] \tag{22}$$

$$[v_L(q)] = [v_n(q)] - \begin{bmatrix} 10\ \Omega & 0 \\ 0 & 10\ \Omega \end{bmatrix} [i_L(q)] \tag{23}$$

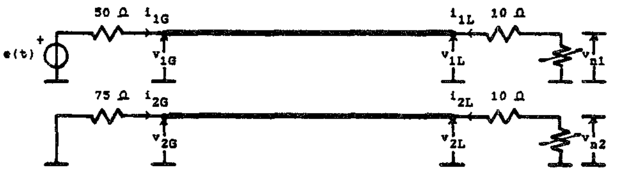

Fig. 3. Schematic representation of the analyzed transmission line with two signal conductors, driven by a voltage generator and terminated in nonlinear loads.

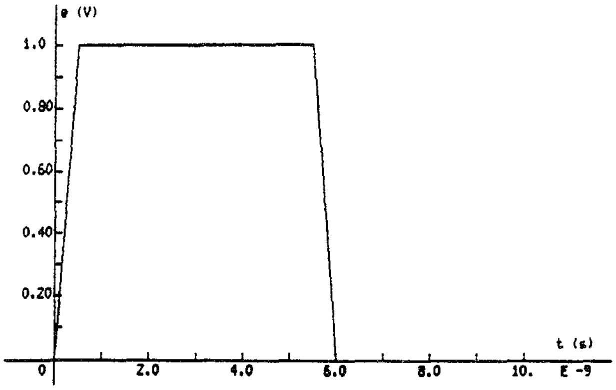

Fig. 4. The waveform $e(t)$ of the voltage generator of Fig. 3.

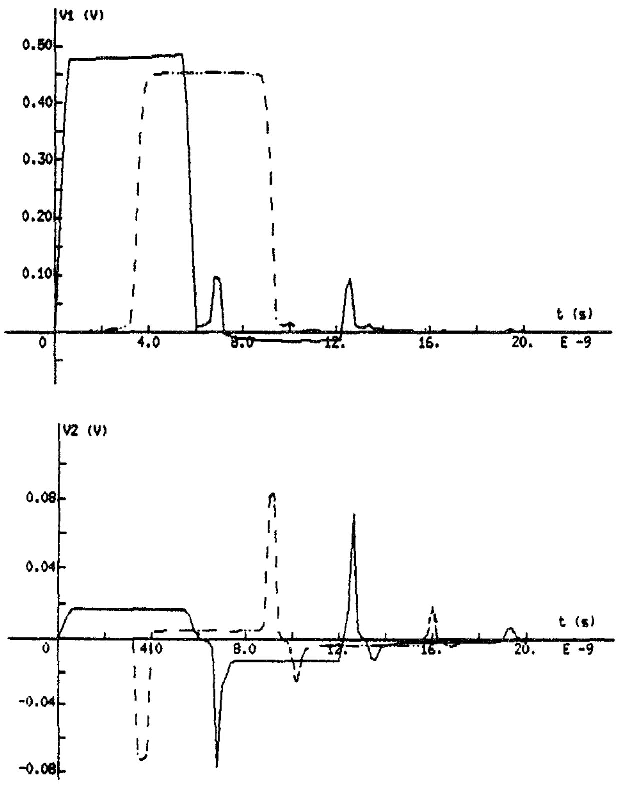

Fig. 5. Response of the transmission line of Fig. 3. —— voltages at the generator end. ---- voltages at the load end.

where the vector $[v_n(q)]$ comprises voltages v_{n1} and v_{n2} at the nonlinear resistors. These voltages are related to the conductor currents through (21), where i_n should be replaced by $-i_{1L}(q)$ and $-i_{2L}(q)$, respectively. The equations for the load end essentially form a system of simultaneous nonlinear equations. These equations were solved here by using the Nelder–Mead simplex algorithm for nonlinear optimization [14].

The voltages at the generator and load ends are given in Fig. 5. In this computation, the time step was taken to be 20 ps, i.e., the highest frequency involved in the computation of Green's functions was 25 GHz. The lengths of the registers containing Green's functions were 512 samples, i.e., 10.24 ns. The CPU time on a VAX 750 computer was about 4.5 min. As a byproduct, the characteristic impedance matrix of the corresponding lossless line was found to be

$$[Z_c] = \begin{bmatrix} 46.38 & 2.66 \\ 2.66 & 46.38 \end{bmatrix} \Omega.$$

In Fig. 5 one can easily trace the voltage at the driven conductor and the voltage at the parasitic conductor (due to the cross-talk) as the waves reach the load end, get reflected, come back to the generator end, and reach the load end again. The line transit time is about 3.4 ns.

IV. Conclusion

A novel technique was presented for the transient analysis of lossy multiconductor transmission lines with arbitrary nonlinear loads. In this approach we compute time-domain Green's functions of the transmission line terminated in quasi-matched loads. These loads change the properties of the transmission line as seen by the line terminal networks, but these properties are easily restored by inserting complementary networks, with negative elements, into the line terminal networks. The complete response of the system is then obtained by using convolution, yielding the equivalent Thévenin network of the transmission line at any time instant. This equivalent network can easily be incorporated into the time-domain solution of the terminal networks. The solution of nonlinear networks (with or without memory) was considered to be a standard circuit-theory technique, as it was not discussed here.

For the present technique, we need fewer time-domain data points than in other techniques for the same resolution. This, in turn, implies that one needs fewer frequency-domain data points when computing Green's functions, and fewer terms in evaluation of the convolution integrals. This significantly improves the CPU time, as the computations in frequency domain and the evaluation of the convolution are the most time-consuming parts of the analysis. An example was included to illustrate the application of the proposed technique.

Appendix

Let us introduce the vector $[I']$ of currents entering the side "a" of the network $[Z_M]$, the vector $[I'']$ of currents entering the side "a" of the network $[-Z_M]$, and the vector $[I_v]$ of virtual port currents, entering the side "b" of the network $[Z_M]$ and at the same time leaving the side "b" of the network $[-Z_M]$. Let us also introduce the vector $[V']$ of voltages at the side "a" of the network $[Z_M]$, the vector $[V'']$ of voltages at the side "a" of the network $[-Z_M]$, and the vector $[V_v]$ of virtual port voltages at the sides "b" of the networks $[Z_M]$ and $[-Z_M]$.

The Z-parameter equations for the two networks now read

$$[V'] = [Z_a][I'] + [Z_m][I_v] \tag{A1}$$

$$[V_v] = [Z_m][I'] + [Z_b][I_v] \tag{A2}$$

$$[V''] = [-Z_a][I''] - [-Z_m][I_v] \tag{A3}$$

$$[V_v] = [-Z_m][I''] - [-Z_b][I_v]. \tag{A4}$$

Taking into account (6), we can subtract (A4) from (A2), thus obtaining

$$[Z_m]([I'] + [I'']) = [0] \tag{A5}$$

where [0] is a null-vector. From (A5) we have

$$[I'] = -[I'']. \tag{A6}$$

If we substitute (A6) into (A3) and compare with (A1), we finally obtain

$$[V'] = [V'']. \tag{A7}$$

Equations (A6) and (A7) essentially state that the corresponding ports at the sides "a" of the networks $[Z_M]$ and $[-Z_M]$ are merely short circuited, because the currents and the voltages at the ports are identical.

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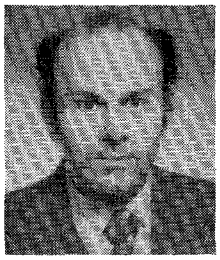

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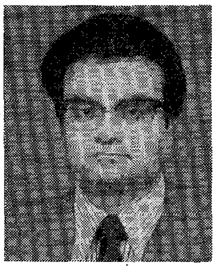

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